UNIT-III

Classification of Random Processes.

Stationary process:

Random Variable:.

A random variable is a rule that assigns a real number to every outcome of a random experiment.

Random process.

A random process is a rule that assigns a time function to every outcome of a random experiment.

A random process is a collection of random variable $\{X(s,t)\}$ $s \in S$ (Sample space) $t \in T$ (Parameter set.

Classification of Random processes.

Discrete Random Sequence.

If both S and T are discrete then the random process is called discrete random sequence.

Eg. No. of books in Library at opening time.

Continuous Random Sequence.

If 'S' is continuous and T is discrete, then the random process is

Eg: Quantity of petrol in the bulk at opening time

Discrete Random process

If `S' is discrete and `T' is Continuous and the random process is called discrete random process.

Eg: No. of phone calls receiving in $(0,t)$

Continuous Random Variable Process

If `S' is continuous and T is continuous, then the random process is called continuous random process.

Eg. Stirring sugar in coffee

Strick Sense Stationary:

A random process is called a Strongly Stationary process (or) Strict Sense Stationary process (SSS) if all its finite dimensional distributions are invariant under translation of time parameter.

Note:

$x(t)$ is SSS

if (i) $E[x(t)]$ is constant

(ii) $E[x^2(t)]$ is Constant.

Wide Sense Stationary:

A random process is called wide sense stationary (WSS) (or) Weakly Stationary process (or) Covariance stationary process.

if (i) $E[X(t)]$ is constant.

(ii) Auto correlation is a function of τ (free from 't') (WSS).

Note:

A random process, non-stationary is called an evalutionary process.

$X(t)$ and $Y(t)$ are said to be Jointly WSS

(i) $R_{XY}(\tau)$ is a function of τ.

if (ii) each process is individually WSS.

22.2.2013.

1. Show that it is not-stationary. The process $X(t)$ whose probability distribution under certain conditions is given by

$$P[X(t)=n\} = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}} & n=1,2,\dots \\ \frac{at}{1+at} & n=0 \end{cases}$$

Solution:

$X(t)=n$	0	1	2	$\dots$

(ii) $E[x^2(t)] = \sum_{n=0}^{\infty} n^2 p(n)$

$$= 0 + \sum_{n=1}^{\infty} n^2 \left(\frac{(at)^{n-1}}{(1+at)^{n+1}}\right)$$

$$= \sum_{n=1}^{\infty} \frac{n^2 (at)^{n-1}}{(1+at)^{n-1+2}}$$

$$= \frac{1}{(1+at)^2} \sum_{n=1}^{\infty} \{n(n+1) - n\} \left(\frac{at}{1+at}\right)^{n-1}$$

$$= \frac{1}{(1+at)^2} \sum_{n=1}^{\infty} n(n+1) \left(\frac{at}{1+at}\right)^{n-1} - \sum_{n=1}^{\infty} n \left(\frac{at}{1+at}\right)^{n-1}$$

$$= \frac{1}{(1+at)^2} \left[\left[1 \cdot 2 \left(\frac{at}{1+at}\right)^0 + 2 \cdot 3 \left(\frac{at}{1+at}\right)^1 + \dots\right] - \left[1\left(\frac{at}{1+at}\right)^0 + 2\left(\frac{at}{1+at}\right) + 3\left(\frac{at}{1+at}\right)^2 + \dots\right]\right]$$

$$= \frac{1}{(1+at)^2} \left[\left[2\left(1 + 3\left(\frac{at}{1+at}\right) + 6\left(\frac{at}{1+at}\right)^2 + \dots\right] - \left[1 + 2\left(\frac{at}{1+at}\right) + 3\left(\frac{at}{1+at}\right)^2 + \dots\right]\right]$$

$$= \frac{1}{(1+at)^2} \left[\left(2\left(1 - \frac{at}{1+at}\right)^{-3} - \left(1 - \frac{at}{1+at}\right)^{-2}\right]$$

$$= \frac{1}{(1+at)^2} \left[2\left(\frac{1+at-at}{1+at}\right)^{-3} - \left(\frac{1+at-at}{1+at}\right)^{-2}\right]$$

$$1 \left[2 \quad 1 \quad - \quad \frac{1}{\,}\right]$$

Given,

$X(t) = \cos(\lambda t + Y)$

$\varphi(\omega) = E[\cos \omega Y + i \sin \omega Y]$

$\varphi(1) = 0$

$\Rightarrow \varphi(1) = E[\cos Y + i \sin Y] = 0$

$E[\cos Y + i \sin Y] = 0.$

$E[\cos Y] + i E[\sin Y] = 0$

$E[\cos Y] = 0$ & $E[\sin Y] = 0.$

$\varphi(2) = 0$

$\Rightarrow E[\cos 2Y + i \sin 2Y] = 0$

$E[\cos 2Y] + i E[\sin 2Y] = 0$

$E[\cos 2Y] = 0$ & $E[\sin 2Y] = 0.$

(i) $E[X(t)] = E[\cos(\lambda t + Y)]$

$= E[\cos(\lambda t) \cos y - \sin \lambda t \sin y]$

$= \cos \lambda t \, E[\cos Y] + \sin \lambda t \, E[\sin Y]$

$= 0$, constant $\because E[\cos Y] = 0$
$E[\sin Y] = 0$

(ii) $R_{XX}(\tau) = E[X(t) \cdot X(t+\tau)]$

$= E[\cos(\lambda t + y) \cdot \cos(\lambda(t+\tau) + \cdot$

$$= E\Big[\cos\lambda t \;\; \cos\lambda(t+\tau)\;\; \cos^2 y +$$

$$\sin\lambda t \;\; \sin\lambda(t+\tau)\;\; \sin^2 y \cancel{+} -$$

$$\big[\cos\lambda t \;\; \sin\lambda(t+\tau)\;\; \cos y \,\sin y +$$

$$\sin\lambda t \;\; \cos\lambda(t+\tau)\;\; \cos y \;\sin y\big]$$

$$= \cancel{E}\big[\cos\lambda t \;\; \cos\lambda(t+\lambda)\;\; E[\cos^2 y] +$$

$$\sin\lambda t \;\; \sin\lambda(t+\tau)\;\; E[\sin^2 y] +$$

$$\big[\cos\lambda t \;\; \sin\lambda(t+\tau) + \sin\lambda t \;\; \cos\lambda(t+\tau$$

$$E\big[\cos y \;\; \sin y\big]$$

$$= \cos\lambda t \;\; \cos\lambda(t+\tau) \;\; E\left[\frac{1+\cos 2y}{2}\right]$$

$$+ \sin\lambda t \;\; \sin\lambda(t+\tau)\;\; E\left[\frac{1-\cos 2y}{2}\right]$$

$$+ E\left[\overset{0}{\frac{\sin 2y}{2}}\right]\Big[\cos\lambda t \;\; \sin\lambda(t+\tau) + \sin\lambda t \;\; \cos\lambda(t+\tau)\Big]$$

$$= \frac{1}{2}\big[\cos\lambda t \;\; \cos\lambda(t+\tau)\big] + \frac{1}{2}\big[\sin\lambda t \;\; \sin\lambda(t+\tau)\big]$$

$$= \frac{1}{2}\big[\cos(\lambda t - \lambda(t+\tau))\big]$$

$$= \frac{1}{2}\big[\cos(\lambda \cancel{t} - \lambda\cancel{t} - \lambda\tau)$$

$$= \frac{1}{2}\cos\lambda\tau$$, free from t.

X(t) is WSS.

4. Show that the process $X(t) = A\cos\lambda t + B\sin\lambda t$. (A, B) are random Variables. is WSS.

(i) $E(A] = E[B] = 0$

(ii) $E[A^2] = E[B^2] = \varphi$

(iii) $E[AB] = 0$.

(i) $E[X(t)] = E[A\cos\lambda t + B\sin\lambda t]$

$$= \cos\lambda t\, E[A] + \sin\lambda t\, E[B].$$

$$= 0, \text{ constant}.$$

(ii) $R_{XX}(\tau) = E[X(t) \cdot X(t+\tau)]$

$$= E\Big[[A\cos\lambda t + B\sin\lambda t]\,(A\cos(\lambda(t+\tau)) + B\sin(\lambda(t+\tau)))\Big]$$

$$= E\Big[A^2\cos\lambda t\cos\lambda(t+\tau) + B^2\sin\lambda t\,\sin\lambda(t+\tau) + AB\cos\lambda t\sin\lambda(t+\tau) + AB\sin\lambda t\cos\lambda(t+\tau)\Big]$$

$$= \cos\lambda t\cos\lambda(t+\tau)\,E[A^2] + \sin\lambda t.\sin\lambda(t+\tau)\,E[B^2] + E[AB]\,[\cos\lambda t\sin\lambda(t+\tau) + \sin\lambda t\sin\lambda(t+\tau)]$$

$$= \cos\lambda t\cos\lambda(t+\tau)\,\varphi +$$

$$= \varphi\left[\cos\lambda t \cos\lambda(t+\tau) + \sin\lambda t \sin\lambda(t+\tau)\right]$$

$$= \varphi\left[\cos(\lambda t - (\lambda(t+\tau))\right]$$

$$= \varphi\left[\cos(\lambda t - \lambda t - \lambda\tau\right]$$

$= \varphi \cos\lambda\tau$, free from t,

X(t) is WSS.

5. Two random processes X(t) and Y(t) are given by

$X(t) = A\cos wt + B\sin wt$

$Y(t) = B\cos wt - A\sin wt$.

Show that, X(t) and Y(t) are jointy WSS, If A and B are uncorrelated r.v with zero mean and the same variances with and w is constant.

Solution:.

$X(t) = A\cos wt + B\sin wt$

$Y(t) = B\cos wt - A\sin wt$.

$E[A] = E[B] = 0$.

$Var(A) = Var(B) = \sigma^2$

$\Rightarrow E[A^2] = E[B^2] = \sigma^2$

A and B are uncorrelated $\Rightarrow E[AB] = 0$.

(i) $X(t)$ is WSS

$$E[X(t)] = E[A\cos\omega t + B\sin\omega t]$$

$$= E[A]\cos\omega t + E[B]\sin\omega t$$

$$= 0, \text{ constant.}$$

$$R_{xx}(\tau) = E[X(t)\cdot X(t+\tau)]$$

$$= E\Big[(A\cos\omega t + B\sin\omega t)\,(A\cos(\omega(t+\tau)) + B\sin(\omega(t+\tau))\Big]$$

$$= E\Big[A^2\cos\omega t\cos(\omega(t+\tau)) + B^2\sin\omega t\overset{\sin}{\cancel{\cos}}(\omega(t+\tau))$$

$$+ AB\big[\cos\omega t\sin(\omega(t+\tau)) + \sin\omega t\cos(\omega(t+\tau))\big]\Big]$$

$$= E[A^2]\cos\omega t\cos(\omega(t+\tau)) + E[B^2]\sin\omega t\sin(\omega(t+\tau))$$

$$+ E[AB]\big[\cos\omega t\sin(\omega(t+\tau)) + \sin\omega t\cos(\omega(t+\tau))\big]$$

$$= \sigma^2\big[\cos\omega t\cos(\omega(t+\tau)) + \sin\omega t\sin(\omega(t+\tau))$$

$$+0$$

$$= \sigma^2(\cos(\omega t - \omega(t+\tau)))$$

$$= \sigma^2(\cos(\cancel{\omega t} - \cancel{\omega t} - \omega\tau))$$

$$= \sigma^2(\cos(-\omega\tau))$$

$$= \sigma^2\cos\omega\tau, \text{ free from } t,$$

$X(t)$ is WSS.

(ii) $Y(t)$ is WSS.

$$E[Y(t)] = E[B \cos wt - A \sin wt].$$

$$= E[B] \cos wt - E[A] \sin wt.$$

$$= 0, \text{ constant},$$

$$R_{YY}(\tau) = E[Y(t).\ Y(t+\tau)]$$

$$= E\Big[(B \cos wt - A \sin wt)(B \cos(w(t+\tau)) - A \sin(w(t+\tau)))\Big]$$

$$= E\Big[B^2 \cos wt \cos(w(t+\tau)) + A^2 \sin wt \sin(w(t+\tau)) - AB \cos wt \sin(w(t+\tau)) - AB \sin wt\ \cancel{\sin}\ \cos(w(t+\tau))\Big]$$

$$= E[B^2] \cos wt \cos(w(t+\tau)) + E[A^2] \sin wt \sin(w(t+\tau)) - E[AB]\Big[\cos wt \sin(w(t+\tau)) + \sin wt \cos(w(t+\tau))\Big]$$

$$= \sigma^2 \Big[\cos wt \cos(w(t+\tau)) + \sin wt \sin(w(t+\tau))\Big]$$

$$= \sigma^2 (\cos (wt - w(t+\tau))]$$

$$= \sigma^2 (\cos(\cancel{wt} - \cancel{wt} - w\tau)]$$

$$= \sigma^2 \{\cos(-w\tau)]$$

$$= \sigma^2 \cos w\tau$$

$R_{YY}(t) = \sigma^2 \cos w\tau$, free from t.

$R_{XY}(\tau) = E[X(t).Y(t+\tau)]$

$= E[(A\cos wt + B\sin wt)(B\cos(w(t+\tau)) - A\sin(w(t+\tau))]$

$= E[B^2\sin wt.\cos(w(t+\tau)) - A^2\cos wt.\sin(w(t+\tau))$

$+ AB$ ~~DAB~~ $\cos wt\cos(w(t+\tau)) -$ ~~AB~~ $(\sin wt\sin(w(t+\tau))$

$= \sigma^2(\sin wt.\cos(w(t+\tau)) - \cos wt\sin w((t+\tau))$

$+ 0$

$= \sigma^2[\sin(wt - wt - w\tau)]$

$= \sigma^2[\sin(-w\tau)]$

$= -\sigma^2\sin w\tau$, free from t,

b. If $X(t) = Y\cos t + Z\sin t$ $\forall t$, where Y and Z are independent binary random variables each of which assumes the values (-1, +2) with Probabilities 2/3 and 1/3 respectively. Prove that X(t) is WSS.

Solution:

Given

$X(t) = Y\cos t + Z\sin t$

Y	-1	2
P(Y)	2/3	1/3

Z	-1	2
P(Z)	2/3	1/3

$= E[Y^2] \cos t \ \cos(t+\tau) + E[Z^2] \sin t \ \sin(t+\tau)$

$+ E[YZ] [\cos t \ \sin(t+\tau) + \sin t \ \cos(t+\tau)]$

$= 2 [\cos t \ \cos(t+\tau) + \sin t \ \sin(t+\tau)] + 0$

$= 2 (\cos(t-t-\tau))$

$= 2 \cos(-\tau)$

$= 2 \cos\tau.$ free from t.

$X(t)$ is WSS.

25/2/2013

Ergodicity:

A random process $X(t)$ is said to be ergodic, if its ensemble averages (Statistical averages (ie) mean, autocorrelation) are equal to appropriate time averages.

If $X(t)$ is a random process, then $\frac{1}{2T}\int_{-T}^{T} X(t)\,dt$ is called time average of $X(t)$ over $(-T, T)$ and denoted by $\overline{X}_T$

$$\overline{X}_T = \frac{1}{2T}\int_{-T}^{T} X(t)\,dt.$$

If the random process $X(t)$ has a constant mean, as $T \to \infty$, then $x(t)$ is said to be mean ergodic.

Problem procedure:

Step 1: Find $\overline{x_T}$

Step 2: Find $E[\overline{x_T}]_T$

Step 3: $Var(\overline{x_T}) = \frac{1}{T}\int_{-T}^{T} C_{xx}(\tau)\left(1-\frac{|\tau|}{T}\right)d\tau$

where

$$C_{xx}(\tau) = E[x(t)\ x(t+\tau)] - E[x(t)]\,E[x(t+\tau)]$$

Step 4: $\lim_{T\to\infty} Var(\overline{x_T}) = 0$

Correlation ergodic:

$x(t)$ is correlation ergodic,

if $\overline{Z}_T = \frac{1}{2T}\int_{-T}^{T} x(t+\tau)\ x(t)\,dt = R(\tau)$

as limit $T \to \infty$

7. If WSS process $x(t)$ is given by $x(t) = 10\cos(100t+\theta)$ where θ is uniformly distributed over $(-\pi,\pi)$. Prove that $x(t)$ is correlation ergodic.

Solution:

$$f(\theta) = \frac{1}{b-a} = \frac{1}{\pi+\pi} = \frac{1}{2\pi}$$

$$R_{xx}(\tau) = E[x(t) \cdot x(t+\tau)]$$

$$= \frac{100}{2} E\left[\cos(200t+100\tau+2\theta) + \cos(100\tau)\right]$$

$$= 50\, E\left[\cos(200t+100\tau+2\theta)\right] + E\left[\cos(100\tau)\right]$$

Consider,

$$E\left[\cos(200t+100\tau+2\theta)\right]$$

$$= \int_{-\pi}^{\pi} \cos(200t+100\tau+2\theta) \cdot \frac{1}{2\pi}\, d\theta$$

$$= \frac{1}{2\pi} \cdot 2 \int_{0}^{\pi} \cos(200t+100\tau+2\theta)\, d\theta$$

$$= \frac{1}{\pi}\left[\frac{\sin(200t+100\tau+2\theta).2}{2}\right]_0^{\pi}$$

$$= 0$$

Sub in (i)

$$R_{xx}(\tau) = 50\{0 + \cos 100\tau\}$$

$$= 50 \cos 100\tau$$

$$\overline{Z}_T = \frac{1}{2T}\int_{-T}^{T} X(t+\tau)\, X(t).\, dt = R(\tau).$$

$$= \frac{1}{2T}\int_{-T}^{T} 10\cos(100t+\theta)\; 10\cos(100(t+\tau)+\theta)\, dt$$

$$= \frac{100}{2T}\int_{-T}^{T} \cos(100t+\theta).\cos(100t+100\tau+\theta)\, dt$$

$$= \frac{50}{T}\int^{T} \frac{1}{2}\left[\cos(200t+100\tau+2\theta) + \cos(-100\tau)\right]$$

$\text{Var}\ \bar{x}_T = 2/3$

$\lim_{T \to \infty} \text{Var}\ \bar{x}_T = \frac{2}{3} \neq 0$

$X(t)$ is not mean ergodic.

Consider 2 random ~~variable~~ process,

$X(t) = 3\cos(\omega t + \theta)$ $\quad Y(t) = 2\cos(\omega t + \theta - \pi/2)$

where θ is a random variable uniformly distributed in $(0, 2\pi)$. Prove that $\sqrt{R_{xx}(0) \cdot R_{yy}(0)} \geq |R_{xy}(\tau)|$

Solution,

Given.

$X(t) = 3\cos(\omega t + \theta)$

$Y(t) = 2\cos(\omega t + \theta - \pi/2)$

θ is uniformly distributed in $(0, 2\pi)$

$$f(\theta) = \frac{1}{b-a} = \frac{1}{2\pi - 0} = \frac{1}{2\pi}$$

$$R_{xx}(\tau) = E[x(t) \cdot x(t+\tau)]$$

$$= E\Big[3\cos(\omega t + \theta) \cdot 3\cos(\omega(t+\tau) + \theta)\Big]$$

$$= 9E\Big[\cos(\omega t + \theta) \cdot \cos(\omega t + \omega\tau + \theta)\Big]$$

$$= \frac{9}{2} E\Big[\cos(2\omega t + \omega\tau + 2\theta) + \cos(-\omega\tau)\Big]$$

$= 9\ E\Big[$

$$= \frac{9}{2} E\Big[\cos(2\omega t + \omega\tau + 2\theta)\Big] + \frac{9}{2} E\Big[\cos(2\omega\tau)\Big]$$

$$= \frac{9}{2} \int_{0}^{2\pi} \cos(2\omega t + \omega\tau + 2\theta) \cdot \frac{1}{2\pi} \, d\theta + \frac{9}{2} \cos\omega\tau$$

$$= \frac{9}{4\pi} \underbrace{\Big[\frac{\sin(2\omega t + \omega\tau + 2\theta)}{2}\Big]_{0}^{2\pi}}_{0} + \frac{9}{2} \cos\omega\tau$$

$$= \frac{9}{2} \cos\omega\tau$$

$$R_{xx}(0) = \frac{9}{2} \cos \omega(0)$$

$$= \frac{9}{2}$$

$$R_{YY}(\tau) = E\Big[Y(t) \cdot Y(t+\tau)\Big]$$

$$= E\Big[2\cos(\omega t + \theta - \pi/2) \cdot 2\cos(\omega t + \omega\tau + \theta - \pi/2)$$

$$= \cancel{\theta}\, 4E\Big[\cos(\omega t + \theta - \pi/2) \cdot \cos(\omega t + \omega\tau + \theta - \pi/$$

$$= \frac{4}{2} E\Big[\cos(2\omega t + \omega\tau + 2\theta - \pi) + \cos(-\omega\tau)\Big]$$

$$= 2E\Big[\cos(2\omega t + \omega\tau + 2\theta - \pi)\Big] + 2E\Big[\cos(\omega\tau)\Big]$$

$$= 2\int_{0}^{2\pi} \cos(2\omega t + \omega\tau + 2\theta + \pi) \frac{1}{2\pi} \, d\theta + 2\cos\omega\tau$$

$$= 2 \Big[\sin(2\omega t + \omega\tau + 2\theta + \pi)\Big]^{2\pi}$$

$R_{YY}(0) = 2\cos w(0)$

$= 2$

$R_{XY}(\tau) = E[X(t).Y(t+\tau)]$

$= E[3\cos(wt+\theta).2\cos(wt+w\tau+\theta-\pi/2)]$

$= 6E[\cos(wt+\theta).\cos(wt+w\tau+\theta-\pi/2)]$

$= 6E[\cos(2wt+w\tau+2\theta-\pi/2)+\cos(-w\tau+\pi/2)]$

$= 6E[\cos(2wt+w\tau+2\theta-\pi/2)]+$

$6E[\cos(\overset{\pi/2-w\tau)}{\cancel{(w\tau-\pi/2)}}))$

$= 6\int_0^{2\pi}\cos(2wt+w\tau+2\theta-\pi/2).\frac{1}{2\pi}\,d\theta$

$+6\cos(\overset{\pi/2-w\tau}{\cancel{w\tau-\pi/2}})$

$= 6E[\cos(wt+\theta).\sin(wt+w\tau+\theta)]$

$= \frac{6}{2}E[\sin(2wt+w\tau+2\theta)+\sin(w\tau)]$

$= \frac{6}{2}E[\sin(2wt+w\tau+2\theta)]+\frac{6}{2}E[\sin w\tau]$

$= \frac{6}{2}\int^{2\pi}\sin(2wt+w\tau+2\theta).\frac{1}{2\pi}\,d\theta+\frac{6}{2}\sin w\tau$

$$= \frac{6}{2}\left[\frac{\cos(2wt + w\tau + 2\theta)}{2 . 2\pi}\right]_0^{2\pi} + \frac{6}{2}\sin w\tau$$

$$= \frac{6}{2}\left[\frac{\cos(2wt + w\tau + 2\pi)}{2 \cdot 2\pi} - \cos(0)\right] + \frac{6}{2}\sin w\tau$$

$$= 3 \sin w\tau$$

$$R_{XY}(\tau) = 3 \sin w\tau$$

$$R_{XX}(0) \cdot R_{YY}(0) = \frac{9}{2} \cdot 2 = 9$$

$$\sqrt{R_{XX}(0) \cdot R_{YY}(0)} = \sqrt{9}$$

$$= 3$$

$$R_{XY}(\tau) = |3 \sin w\tau| \leq 3$$

$$R_{XY}(\tau) \leq \sqrt{R_{XX}(0) \cdot R_{YY}(0)}.$$

Markov process

Future depends only upon the Present but not on past.

If for all n, ~~$P[x_n = a_n / x_{n-1} = a_{n-1}$~~

$$P[x_n = a_n / x_{n-1} = a_{n-1} \dots x_0 = a_0] =$$

$$P[x_n = a_n / x_{n-1} = a_{n-1}]$$

~~of~~ $\{x_n\}$ then the process $\{x_n\}$, $n = 0, 1, 2 \dots$ is called Markov Chain.

(i) $a_1, a_1, \dots a_n$ are called states.

(ii) $P[x_n = a_j / x_{n-1} = a_i]$ is called one step

(iii) $P[x_n = a_j / x_0 = a_i]$ is called 'n' step transistion probability from State a_i to a_j.

Note 1:

The tpm of a margkov change is a stochastic matrix since $P_{ij} \geq 0$ and $\Sigma P_{ij} = 1$ (ie) Sum of all the elements of row of the

Note 2:

A stochastic matrix 'P' is said to be a regular matrix, if all the entries of $\underset{\sim}{P}^{m}$ (Possible integer m) are Positive.

Note 3:

A homogeneous markov chain is said to be regular, if its tpm is regular

Note 4:

If $P_{ij}^{m} > 0$, for some 'n' and ∀ i and j, that every state can be reached from every other state. Here, markov chain is said to be irreduceable.

Note 5:

The period ~~B~~ di of a return state, i is defined as the greatest common divisor of all m, such that $P_{ij}^{m} > 0$. State i is said to be periodic with period di, if di>1, and aperiodic if di=1

Note 6 :

A non-null persistent and Apeariodic state is called ergodic.

Note 7 :

If a markov chain is irreduceable, all its states are of the same time,

ib a markov chain is finite irreduceable, all its states are non-null persistant.

Note 8:

Steady state, probability distribution or Stationary state distribution of the markov chain is $\pi p = \pi$

Note 9 :

To find irreduceable nature ; $P^2, P^3, P^4 \ldots$ and note all $P_{ij} > 0$, at some 'P^n'

To find period type ; called the Powers of P, where $P_{ii} > 0$, and find gcd of powers

To find steady state ; find $\pi p = \pi$

Find the nature of the states of the markov chain if the tpm

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \longrightarrow \text{future.} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

↓
Present

Solution,

Given.

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P^2 = P \cdot P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

$$P^3 = P \cdot P^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P^4 = P \cdot P^3 = \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

Markov chain is introducable and finite.

⇒ All states are non-null Persistant.

i) $P_{11}^{(2)} > 0$, $P_{11}^{(4)} > 0$

⇒ gcd $\{2, 4 \ldots\} = 2$

⇒ state (0 is Period 2)

$P_{22}^{(2)} > 0$, $P_{\cancel{33}\,22}^{(4)} > 0$

gcd $\{2, 4, \ldots\} = 2$

⇒ state 1 is Period 2

$P_{33}^{(2)} > 0$, $P_{33}^{(4)} > 0 \ldots$

gcd $\{2, 4 \ldots\} = 2$

⇒ State 2 is period 2.

Here all states are periodic with period 2.

Here all states are non-null persistent and periodic

⇒ All states are ergodic